

## Subiectul I

$$\textcircled{1} \quad \overline{17x} : 10 \Leftrightarrow x = 0 \Rightarrow \overline{17x} = 170$$

d)

$$\textcircled{2} \quad 20\% \text{ din } 50 = \frac{20}{100} \cdot 50 = \frac{1}{5} \cdot 50 = \frac{50}{5} = 10$$

a)

$$\textcircled{3} \quad [-2, 3] \cap \mathbb{Z} = \{-2, -1, 0, 1, 2, 3\}$$

$$S = -2 + (-1) + 0 + 1 + 2 + 3 = 3$$

c)

$$\textcircled{4} \quad \left(\frac{2}{3}\right)^{-1} = \frac{3}{2}$$

d)

$$\textcircled{5} \quad m_g(x, y) = \sqrt{x \cdot y} = \sqrt{(3-2\sqrt{2})(3+2\sqrt{2})} = \sqrt{9-8} = \sqrt{1} = 1$$

c)

$$\textcircled{6} \quad 4 = \sqrt{16} \quad | \quad 2\sqrt{5} = \sqrt{2^2 \cdot 5} = \sqrt{20} \quad | \quad \Rightarrow 4 < 2\sqrt{5} \quad \text{Afirmația e falsă}$$

b)

## Subiectul II

①  $C = \sin_A B \Rightarrow A$ - mijlocul lui  $BC \Rightarrow BC = 2 \cdot AB = 2 \cdot 5 = 10 \text{ cm}$   
 $D = \sin_B C \Rightarrow B$ - mijlocul lui  $CD \Rightarrow CD = 2 \cdot BC = 2 \cdot 10 = 20 \text{ cm}$

d)

②  $\angle AOC + \angle COB = \angle AOB \Rightarrow 30^\circ + \angle COB = 180^\circ \Rightarrow \angle COB = 150^\circ$

OE - bis.  $\angle COB \Rightarrow \angle EOB = \frac{\angle COB}{2} = \frac{150^\circ}{2} = 75^\circ$

$\angle AOC \equiv \angle BOD$  (op. v.f.)  $\Rightarrow \angle BOD = 30^\circ$

$\angle DOE = \angle DOB + \angle BOE = 30^\circ + 75^\circ = 105^\circ$

c)

③ M, N, P - mijloacele AB, BC, AC  $\Rightarrow P_{MNP} = \frac{P_{ABC}}{2} =$   
 $= \frac{AB + BC + AC}{2} = \frac{12 + 13 + 7}{2} = \frac{32}{2} = 16 \text{ cm}$

b)

④ M - mijlocul lui AB  $\Rightarrow \left\{ \begin{array}{l} DM - \text{mediană în } \triangle ABD \\ CM - \text{mediană în } \triangle ABC \end{array} \right. \Bigg| \begin{array}{l} \text{T. med.} \\ \Rightarrow \end{array}$

$\left\{ \begin{array}{l} AC \perp BC \\ AD \perp BD \end{array} \right. \Rightarrow \left\{ \begin{array}{l} \triangle ABC - \text{dreptunghiic în } C \\ \triangle ABD - \text{dreptunghiic în } D \end{array} \right.$

$\left\{ \begin{array}{l} DM = \frac{AB}{2} \\ CM = \frac{AB}{2} \end{array} \right. \Rightarrow DM \equiv CM \Rightarrow \triangle DMC - \text{isoscel cu baza } DC$

$\Rightarrow \angle MDC = \angle MCD = 40^\circ$ . În  $\triangle DMC$ ,  $\angle MDC + \angle MCD + \angle CMD = 180^\circ$

$\Rightarrow 40^\circ + 40^\circ + \angle CMD = 180^\circ \Rightarrow \angle CMD = 100^\circ$

c)

⑤ BC - diameter  $\Rightarrow \widehat{BC} = 180^\circ$

$$\widehat{AB} = \widehat{BC} - \widehat{AC} = 180^\circ - 120^\circ = 60^\circ \Rightarrow \angle AEB = \frac{\widehat{AB}}{2} = \frac{60^\circ}{2} = 30^\circ$$

a)

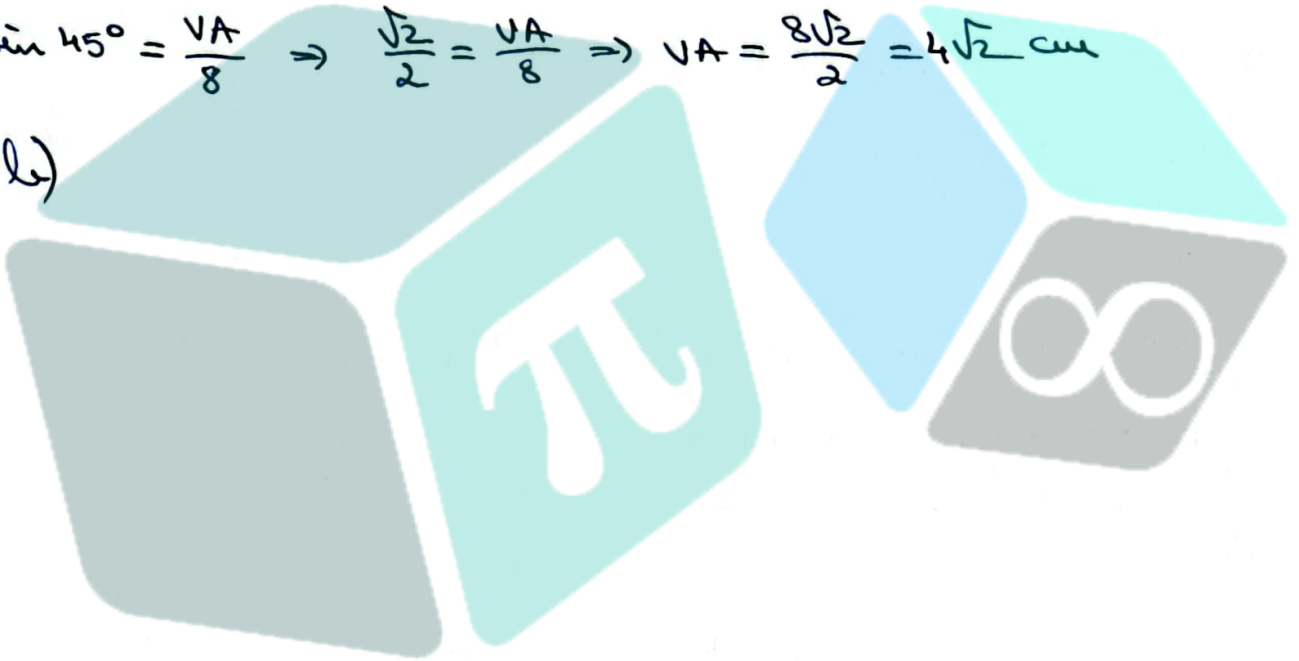
⑥  $VA \equiv VB \Rightarrow \triangle VAB$  - isoscel cu baza  $AB$   $\mid \Rightarrow \angle VBA = 45^\circ$   
 $\triangle VAB$  dreptunghic

$$AB = 2 \cdot AO = 2 \cdot 4 = 8 \text{ cm}$$

În  $\triangle VAB$  dreptunghic în  $V$ ,  $\sin \angle VBA = \frac{VA}{AB} \Rightarrow$

$$\sin 45^\circ = \frac{VA}{8} \Rightarrow \frac{\sqrt{2}}{2} = \frac{VA}{8} \Rightarrow VA = \frac{8\sqrt{2}}{2} = 4\sqrt{2} \text{ cm}$$

b)



hei profu'

### Subiectul III

- ① a) Presupunem că sunt 16 apartamente cu 4 camere.  
Ele au, în total,  $16 \cdot 4 = 64$  camere  $> 60$ . Contradicție!  
Deci nu e posibil să fie 16 ap. cu 4 cam.

b) Fie  $\begin{cases} x - \text{nr. ap. cu 2 cam.} \\ y - \text{nr. ap. cu 4 cam.} \end{cases} \Rightarrow$

$$\begin{cases} x + y = 22 \quad | \cdot 4 \\ 2x + 4y = 60 \end{cases} \Rightarrow \begin{cases} 4x + 4y = 88 \\ 2x + 4y = 60 \end{cases} \stackrel{(-)}{\Rightarrow} 2x = 28 \quad | : 2$$

$$\Rightarrow x = 14.$$

Sunt 14 apartamente cu două camere în bloc

② a)  $E(x) = \left( \frac{x^2 - 9}{x^2 - 16} - 1 \right) : \left( \frac{1}{x+4} + \frac{1}{x-4} - \frac{3}{x^2 - 16} \right), x \in \mathbb{R} \setminus \left\{ -4, 4, \frac{3}{2} \right\}$

$$\Rightarrow E(x) = \left( \frac{x^2 - 9}{x^2 - 16} - \frac{x^2 - 16}{x^2 - 16} \right) : \left( \frac{x-4}{x^2 - 16} + \frac{x+4}{x^2 - 16} - \frac{3}{x^2 - 16} \right) =$$

$$= \frac{7}{x^2 - 16} : \frac{2x - 3}{x^2 - 16} = \frac{7}{x^2 - 16} \cdot \frac{x^2 - 16}{2x - 3} = \frac{7}{2x - 3}, \forall x \in \mathbb{R} \setminus \left\{ -4, 4, \frac{3}{2} \right\}$$

b) Fie  $m \in \mathbb{N}$  a. i.  $E(m) \in \mathbb{N} \Rightarrow \frac{7}{2m - 3} \in \mathbb{N} \Leftrightarrow$

$$\begin{cases} m \in \mathbb{N} \cap \left( \mathbb{R} \setminus \left\{ -4, 4, \frac{3}{2} \right\} \right) \\ 2m - 3 \in \mathbb{N} \\ 2m - 3 \mid 7 \end{cases} \Leftrightarrow 2m - 3 \in \{1, 7\} \Leftrightarrow$$

$$2m \in \{4, 10\} \Leftrightarrow m \in \{2, 5\}$$



$$\textcircled{3} \text{ a) } a = \left(-\frac{1}{3}\right)^{32} : \left(-\frac{1}{3}\right)^{30} \cdot (-6)^2 = \left(-\frac{1}{3}\right)^2 \cdot 36 = \left(\frac{1}{3}\right)^2 \cdot 36 =$$

$$= \frac{1}{9} \cdot 36 = \frac{36}{9} = 4$$

$$\text{b) } b = \left(\frac{1}{1+2} + \frac{1}{1+2+3}\right) \cdot (0,5)^{-2} = \left(\frac{1}{3} + \frac{1}{6}\right) \cdot \left(\frac{1}{2}\right)^{-2} =$$

$$= \left(\frac{2}{6} + \frac{1}{6}\right) \cdot 2^2 = \frac{3}{6} \cdot 4 = \frac{1}{2} \cdot 4 = \frac{4}{2} = 2$$

$$m_a(a, b) = \frac{a+b}{2} = \frac{4+2}{2} = \frac{6}{2} = 3$$

$$\textcircled{4} \text{ a) } A_{ABC} = \frac{BC \cdot AC \cdot \sin \hat{ACB}}{2} = \frac{10 \cdot 20 \cdot \sin 30^\circ}{2} = 100 \cdot \frac{1}{2} = 50 \text{ cm}^2$$

$$\text{b) } \begin{array}{l} \triangle ABC \\ \triangle BDC \end{array} \left| \begin{array}{l} \hat{BAC} \equiv \hat{DBC} \\ \hat{C} - \text{comun} \end{array} \right| \begin{array}{l} \text{U.U.} \\ \Rightarrow \end{array} \triangle ABC \sim \triangle BDC \Rightarrow$$

$$\frac{AC}{BC} = \frac{BC}{DC} \Rightarrow \frac{20}{10} = \frac{10}{DC} \Rightarrow DC = \frac{10 \cdot 10}{20} = \frac{100}{20} = 5 \text{ cm}$$

$$\textcircled{5} \text{ a) } \triangle ABE - \text{dreptunghic} \left| \Rightarrow \hat{BAE} = 45^\circ \right.$$

$$AB \equiv BE$$

$$\hat{DAE} = \hat{DAB} + \hat{BAE} = 45^\circ + 45^\circ = 90^\circ \Rightarrow DA \perp AE$$

$$\text{b) } \text{Fie } CB \cap AE = \{M\}$$

$$ABCD - \text{romb} \Rightarrow CB \parallel AD \left| \Rightarrow CB \perp AE \Rightarrow CM \perp AE \Rightarrow \right.$$

$$AD \perp AE$$

$$\triangle CMA \text{ dreptunghic in } M$$

$$ABCD - \text{romb} \Rightarrow BC = AB \Rightarrow BC = 10 \text{ cm}$$

$$\triangle ABE - \text{dreptunghic in } B \xrightarrow{\text{T.Pit.}} AE^2 = AB^2 + BE^2 \Rightarrow AE^2 = 10^2 + 10^2$$

$$\Rightarrow AE^2 = 200 \Rightarrow AE = 10\sqrt{2}$$

$$BM \perp AE, M \in AE \Rightarrow BM - \text{inaltime in } \triangle ABE, \text{ital cu baza } AE$$

$$\Rightarrow BM - \text{mediană în } \triangle ABE \quad \left| \begin{array}{l} \text{T. med.} \\ \Rightarrow \end{array} \right. \quad BM = \frac{AE}{2} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$$

$\triangle ABE - \text{dreptunghic în } B$

$$CM = BC + BM = 10 + 5\sqrt{2} \text{ cm}$$

$$BM - \text{mediană în } \triangle ABE \Rightarrow M - \text{mijlocul lui } AE \Rightarrow AM = \frac{AE}{2} = \frac{10\sqrt{2}}{2} = 5\sqrt{2} \text{ cm}$$

În  $\triangle ACM$  dreptunghic în  $M$  avem

$$\begin{aligned} \operatorname{tg} \angle CAM &= \frac{CM}{AM} \Rightarrow \operatorname{tg} \angle CAE = \frac{10 + 5\sqrt{2}}{5\sqrt{2}} = \frac{10}{5\sqrt{2}} + \frac{5\sqrt{2}}{5\sqrt{2}} = \\ &= \frac{\sqrt{2}}{2} + 1 = \frac{2\sqrt{2}}{2} + 1 = \sqrt{2} + 1 \end{aligned}$$

⑥ a)  $AC - \text{diagonală a feței } ABCD \quad \left| \Rightarrow AC = 6\sqrt{2} \text{ cm} \right.$   
 $ABCD A'B'C'D' - \text{cub, } AB = 6 \text{ cm}$

$$CP = AC - AP = 6\sqrt{2} - 2\sqrt{2} = 4\sqrt{2} = 2 \cdot 2\sqrt{2} = 2 \cdot AP$$

b)  $M - \text{mijlocul lui } B'C' \Rightarrow B'M = \frac{B'C'}{2} = \frac{6}{2} = 3 \text{ cm}$

$$B'C' \parallel BC \Rightarrow B'M \parallel BC \xrightarrow[\triangle BRC]{\text{T.F.A.}} \triangle MRB' \sim \triangle BRC \Rightarrow$$

$$\begin{aligned} \frac{B'R}{RC} = \frac{MB'}{BC} &\Rightarrow \frac{B'R}{RC} = \frac{3}{6} = \frac{1}{2} \quad \left| \Rightarrow \frac{B'R}{RC} = \frac{AP}{PC} \right. \quad \begin{array}{l} \text{R.T. Thales} \\ \xrightarrow{\triangle ACB'} \end{array} \\ CP = 2 \cdot AP &\Rightarrow \frac{AP}{CP} = \frac{1}{2} \end{aligned}$$

$$PR \parallel AB' \Rightarrow \angle (PR, AD') = \angle (AB', AD')$$

$AD', AB', B'D' - \text{diagonale ale fețelor cubului} \Rightarrow$

$$AD' \equiv AB' \equiv B'D' \Rightarrow \triangle AB'D' - \text{echilateral} \Rightarrow \angle B'AD' = 60^\circ$$

$$\Rightarrow \angle (AB', AD') = 60^\circ \Rightarrow \angle (PR, AD') = 60^\circ$$